How to assess the fit of choice models with Stata?

2024 German Stata Conference
at GESIS in Mannheim,
June 7, 2024

"There is no safety in numbers." Howard S. Wainer

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Outline

- **What is the problem?**
- **What is the solution in Stata?**
- **Example of application**
- **Conclusions**

What is the problem?

- **In 1992 Stata V3 introduced the clogit-command to estimate Conditional (fixed-effects) logistic regression model which calculates the McFadden Pseudo R^2**
- **In 2007 Stata V10 introduced the asclogit-command to estimate the alternative-specific conditional logit model**
- **In 2019 Stata V16 introduced the Choice Models (cm) commands**
- **But none of them calculates the Likelihood-Ratio-chi² test statistic and any Pseudo R^2 to assess the fit of the model !**

What is the solution in Stata?

- **My fit_cmclogit.ado calculates for McFadden's conditional logit choice model the following test statistic and Pseudo R^2s tested by Monte Carlo simulation studies in the 1990s / 2000s**
  - **Likelihood-Ratio-χ^2 test statistic using a zero model with alternative-specific constants**
  - **McFadden Pseudo R^2 (likelihood-ratio-index) (1974)**
  - **Adjusted McFadden Pseudo R^2 (1985)**
  - **Maddala Pseudo R^2 (1983)**
  - **Cragg & Uhler Pseudo R^2 (1970)**
  - **Aldrich & Nelson Pseudo R^2 (1984)**
  - **Aldrich & Nelson Pseudo R^2 with Veall & Zimmermann correction (1994)**

Example of application

- **North Rhine-Westphalia Election Study of 1995**
  - **CATI Survey with 504 respondents (eligible voters)**
  - **Endogenous variable: voting intention for the German parties SPD, FDP or CDU: 1) yes 0) no**
  - **Exogenous variables**
    - **Generic / alternative specific: long term preference for one of the three parties (gprefall): 1) yes 0) no**
    - **Case-specific variables:**
      - **Religious denomination (confession): 1) yes 0) no**
      - **Educational degree (education): 1) secondary modern 2) secondary modern+ 3) grammar school 4) college/university**
  - **Balanced hierarchical data structure**
    - **Party alternatives are nested within respondents**

Stata 18 Output

```
Conditional logit choice model                   Number of obs      =      1,512
Case ID variable: probnr                         Number of cases    =        504

Alternatives variable: party                     Alts per case: min =          3
                                                                avg =        3.0
                                                                max =          3

                                                    Wald chi2(9)    =     263.97
Log likelihood = -259.67913                         Prob > chi2     =     0.0000

------------------------------------------------------------------------------------
              vote | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------------+----------------------------------------------------------------
party              |
          gprefall |
               yes |   2.193726   .1401447    15.65   0.000     1.919048    2.468405
-------------------+----------------------------------------------------------------
SPD                |
        confession |
               yes |  -.9000949   .3084457    -2.92   0.004    -1.504637   -.2955524
                   |
         education |
       sec.modern+ |  -.1846034   .3412324    -0.54   0.589    -.8534067    .4841998
    grammar school |   -.645902   .5506053    -1.17   0.241    -1.725069    .4332646
college/university |   -1.03819   .6887728    -1.51   0.132     -2.38816    .3117801
                   |
             _cons |   .4353825   .2737489     1.59   0.112    -.1011554    .9719205
-------------------+----------------------------------------------------------------
FDP                |
        confession |
               yes |  -.6455168   .3947333    -1.64   0.102     -1.41918    .1281462
                   |
         education |
       sec.modern+ |   1.393966   .4604399     3.03   0.002     .4915205    2.296412
    grammar school |   2.076665   .6434303     3.23   0.001     .8155643    3.337765
college/university |   3.160799   .5990928     5.28   0.000     1.986598    4.334999
                   |
             _cons |  -1.956077   .3772011    -5.19   0.000    -2.695377   -1.216776
-------------------+----------------------------------------------------------------
CDU                |  (base alternative)
------------------------------------------------------------------------------------
```

Output of my fit_cmclogit.ado

```
. fit_cmclogit

Likelihood-Ratio-chi2 test against zero model with ASCs

H0: all alternative-/case-specific-effects are zero in the population

LR chi2(  9) =    463.32    Prob > chi2 = 0.0000

Fit-Indices for the Alternative-Specific-Conditional-Logit model:

McFadden Pseudo R2 (compared with zero model with ASCs) = 0.4715

McFadden Pseudo R2 with Ben-Akiva & Lerman correction = 0.4532

Maddala ML Pseudo R2 = 0.6012

Cragg & Uhler Pseudo R2 = 0.7009

Aldrich & Nelson Pseudo R2 = 0.4790

Aldrich & Nelson Pseudo R2 with Veall & Zimmermann correction = 0.7246
```

Excellent fit!

My ado returns the following r-containers

```
. return list

scalars:
                 r(logl_m0) =  -491.3376899127339
                 r(logl_ma) =  -259.679126768394 8
               r(an_pr2_vz) =  .724628733565413 9
                  r(an_pr2) =  .478971284284291 3
                  r(cu_pr2) =  .700944868912334 4
                  r(ml_pr2) =  .601193926861091 2
                r(rho2_bar) =  .453168091346473 5
                    r(rho2) =  .471485432321472 8
                    r(lr_p) =  0
                   r(lr_df) =  9
                 r(lr_chi2) =  463.317126288678 2
```

Conclusions

- **What have I shown?**
  - **My fit_cmclogit.ado allows to assess the fit of McFadden's choice model in a user-friendly way.**
  - **It provides all information we need to evaluate the model fit.**
- **What's in progress?**
  - **Following extensions are in the pipeline**
    - **Construction of McFadden's prediction-success table**
    - **Calculation of a separate McKelvey & Zavoina Pseudo R^2 for each logit equation**

Closing words

- **Thank you for your attention**
- **Do you have some questions?**

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Appendix

What is the solution?

- **Short review of the Monte-Carlo studies made by econometricians to test systematically the most common Pseudo R^2s for binary and ordinal probit / logit models**
  - **Hagle & Mitchell 1992**
  - **Veall & Zimmermann 1992, 1993, 1994**
  - **Windmeijer 1995**
  - **DeMaris 2002**

- **My fit_cmclogit.ado to calculate the most important Pseudo-R^2s**

Which Pseudo-R^2s were tested in the MC studies?

- **Likelihood-based measures:**
  - **Maddala / Cox & Snell Pseudo R^2 (1983/1989)**
  - **Cragg & Uhler / Nagelkerke Pseudo R^2 (1970/1992)**
- **Log-Likelihood-based measures:**
  - **McFadden Pseudo R^2 (1974)**
  - **Aldrich & Nelson Pseudo R^2 (1984)**
  - **Aldrich & Nelson Pseudo R^2 with the Veall & Zimmermann correction (1992)**
- **Basing on the estimated probabilities:**
  - **Efron / Lave Pseudo R^2 (1970 / 1978)**
- **Basing on the variance decomposition of the estimated Probits / Logits:**
  - **McKelvey & Zavoina Pseudo R^2 (1975)**

Results of the Monte-Carlo-studies for binary / ordinal logits or probits

- **The McKelvey & Zavoina Pseudo R^2 is the best estimator for the "true R^2" of the OLS regression**
- **The Aldrich & Nelson Pseudo R^2 with the Veall & Zimmermann correction is the best approximation of the McKelvey & Zavoina Pseudo R^2**
- **Lave / Efron, Aldrich & Nelson, McFadden and Cragg & Uhler Pseudo R^2 underestimate the "true R^2" of the OLS regression**
- **My personal advice: Use the McKelvey & Zavoina Pseudo R^2 or the Aldrich & Nelson Pseudo R^2 with Veall & Zimmermann correction to assess the fit of binary and ordinal logit models**

Log-Likelihood-based measures 1

- **McFadden-Pseudo-R^2 (1974) provided by Stata**

$$McFadden\ Pseudo\ R^2\left(\rho^2\right) = 1 - \left[\frac{\log L_A}{\log L_0}\right]$$

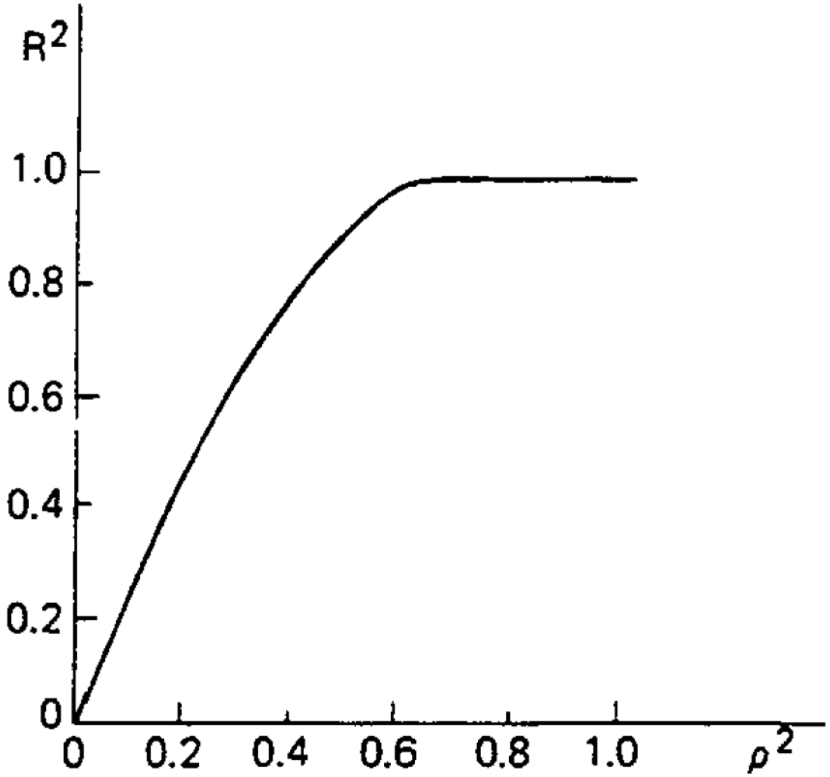
Theoretical range: $0 \leq$ McFadden Pseudo $R^2 \leq 1$

but ρ^2 does not reach its maximum of one!

Rule of thumb: $0.20 \leq$ McFadden Pseudo $R^2 \leq 0.40$ marks an excellent fit. It is equivalent to $0.40 \leq R^2 \leq 0.80$ of a linear regression model (McFadden 1978: 307)

Legend: $\log L_A$: Log-Likelihood of alternative model
$\log L_0$: Log-Likelihood of zero model

Relationship between McFadden's ρ^2 and R^2 of the regression model



(Figure 5.5 in Domencich & McFadden 1975: 124)

Interpretation

„Those unfamiliar with the *p2* index should be forewarned that its values tend to be considerably lower than those of the R2 index and should not be judged by the standards for a 'good fit' in ordinary regression analysis. For example, values of 0.2 to 0.4 for *p2* represent an excellent fit."

(McFadden 1978: 307)

Log-Likelihood-based measures 2

- **Adjusted McFadden Pseudo R^2 (1985)**

$$McFadden\ Pseudo\ R^2_{adjusted}\left(\bar{\rho}^2\right) = 1 - \left[\frac{\log L_A - K}{\log L_0}\right]$$

Correction of McFadden Pseudo R² by the total number of estimated logistic slopes (K) proposed by Ben-Akiva & Lerman (1985: 167)

Range: $0 \leq$ McFadden Pseudo $R^2_{adjusted} \leq 1$,
but it does not reach its maximum of one!

Likelihood-based measures 1

- **Maddala Pseudo-R^2 (1983) or Cox & Snell Pseudo R^2 (1989):**

$$Maddala\ Pseudo\ R^2 (R^2_{ML}) = 1 - \left[\frac{L_0}{L_A} \right]^{\frac{2}{n}}$$

$$= 1 - \exp\left(\frac{-L.R.\chi^2}{n} \right) = 1 - \exp\left(\frac{-2 \times \left(\log L_A - \log L_0 \right)}{n} \right)$$

$$Range: \quad 0 \leq Maddala\ Pseudo\ R^2 \leq 1 - L_0^{\frac{2}{n}}$$

Legend:
L_0 : Likelihood of zero model (constant only)
L_A : Likelihood of alternative model
n : number of cases

Likelihood-based measures 2

- **Cragg & Uhler Pseudo R² (1970) or Nagelkerke Pseudo R² (1991)**

$$Cragg\ \&\ Uhler\ Pseudo\ R^2 = \frac{R^2_{ML}}{\max .R^2_{ML}}$$

$$= \frac{1 - \left[\frac{L_0}{L_A}\right]^{\frac{2}{n}}}{1 - L_0^{\frac{2}{n}}} = \frac{1 - \exp\left(\frac{-L.R.\chi^2}{n}\right)}{1 - \exp\left(\frac{2}{n} \times \log L_0\right)}$$

Correction of the Maddala Pseudo R^2 by its own theoretical maximum➜ Range: $0 \leq$ C&U Pseudo $R^2 \leq 1$

Legend: log: Logarithmus naturalis
exp: Exponential function

Log-Likelihood-based measures 3

- **Aldrich & Nelson Pseudo R^2 (1984)**

$$Aldrich\ \&\ Nelson\ Pseudo\ R^2 = \frac{L.R.\chi^2}{L.R.\chi^2 + n}$$

$$= \frac{2 \times (\log L_A - \log L_0)}{2 \times (\log L_A - \log L_0) + n}$$

Veall & Zimmermann Correction

- **Veall & Zimmermann (1994) propose a correction of the Aldrich & Nelson Pseudo R^2 by its upper limit**
  - **Range of the A&N Pseudo R^2**

$$0 \leq A\,\&\,N\ Pseudo\ R^2 \leq \frac{-2 \times \log L_0}{n - 2 \times \log L_0}$$

  - **Correction formula**

$$A\,\&\,N\ Pseudo\ R^2_{V\,\&\,Z} = \frac{\dfrac{2 \times \left(\log L_A - \log L_o\right)}{2 \times \left(\log L_A - \log L_o\right) + n}}{\dfrac{-2 \times \log L_0}{n - 2 \times \log L_0}}$$

Basing on the estimated probabilities

- **Lave / Efron Pseudo R^2 (1979)**

$$Lave\,/\,Efron\ Pseudo\ R^2 = 1 - \frac{\sum_{i=1}^{n}\left(Y_i - \hat{p}_i\right)^2}{\sum_{i=1}^{n}\left(Y_i - \overline{Y}\right)^2}$$

with $\overline{Y} = \frac{1}{n} \times \sum_{i=1}^{n} Y_i$

Legend:

Y_i: Value of the dependent variable for case i (1 or 0)

$\hat{p}_i$: Estimated probability Y=1 for case i

$\overline{Y}$: Relative frequency of Y=1

Variance decomposition of estimated Y*

- **McKelvey & Zavoina Pseudo R^2 (M&Z Pseudo R^2)**

$$M\ \&\ Z\ Pseudo\ R^2 = \frac{Var\left(\hat{y}^*\right)}{Var\left(\hat{y}^*\right)+Var\left(\varepsilon\right)} = \frac{\dfrac{\sum_{i=1}^{n}\left(\hat{y}_i^* - \overline{\hat{y}^*}\right)^2}{n}}{\dfrac{\sum_{i=1}^{n}\left(\hat{y}_i^* - \overline{\hat{y}^*}\right)^2}{n} + n \times \frac{\pi^2}{3}}$$

Range: $0 \leq$ M&Z Pseudo $R^2 \leq 1$

Legend:

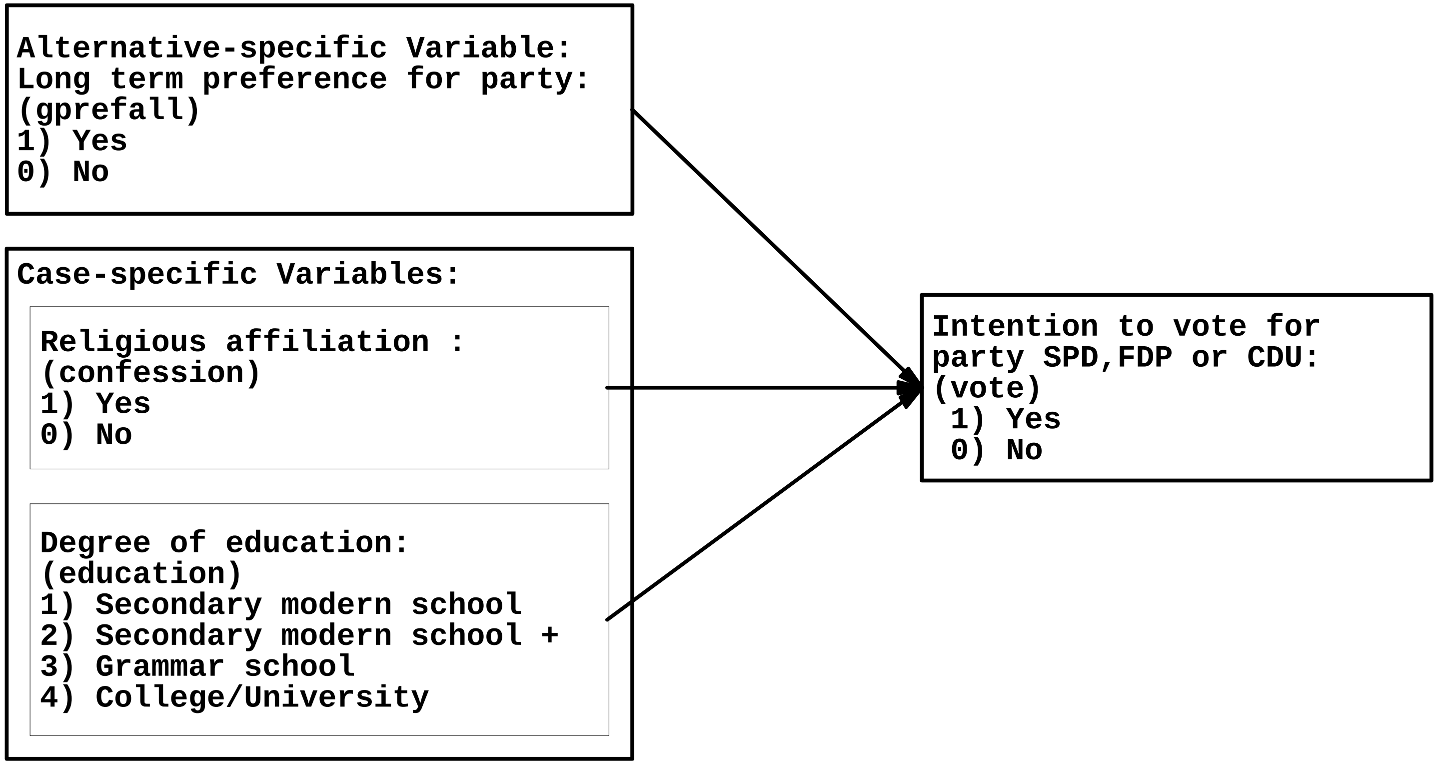
$Var\left(\hat{y}^*\right)$: Variance of the estimated logits (latent variable Y^*)

$\hat{y}_i^*$: Estimated logit of case i

$\overline{\hat{y}^*}$: Mean of the estimated logits

$\frac{\pi^2}{3}$: Variance of logistic density function

Theoretical Model



- **Reference group: respondents with secondary modern degree, without religious affiliation and no party preference**

McFadden's choice model (cmclogit)

- **Estimation equation**

$$\ln\left[\frac{P_i(Y=j)}{P_i(Y=J)}\right] = \sum_{k=1}^{K} \gamma_k \times (z_{ijk} - z_{iJk}) + \sum_{j}^{J-1} \alpha_j + \sum_{l=1}^{L} ß_l \times X_{il}$$

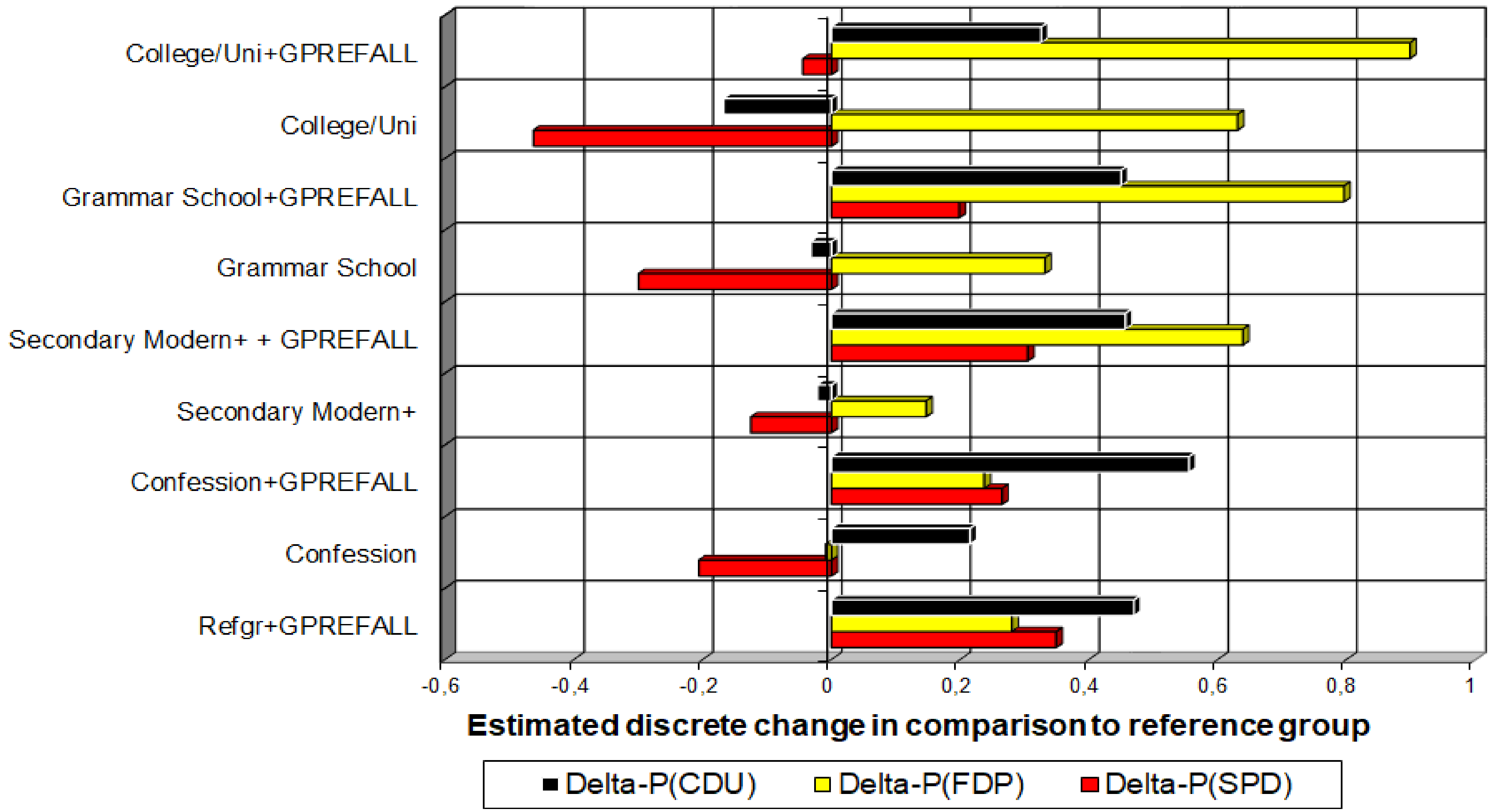
Rational Choice-part with alternative-specific γ-logit slopes for the difference of Z_k

Multinomial logit model to estimate the effects case-specific exogenous variables

β-logistic slope of the effect of X_l on comparison j vs. J

Logistic constant for the comparison j vs. J

Estimated effects of exogenous variables



Reference group: **P(CDU)=0.3722** **P(FDP) = 0.0526** **P(SPD)= 0.5751**