Heterogeneous Difference-in-Differences in Stata

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Stata

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Introduction

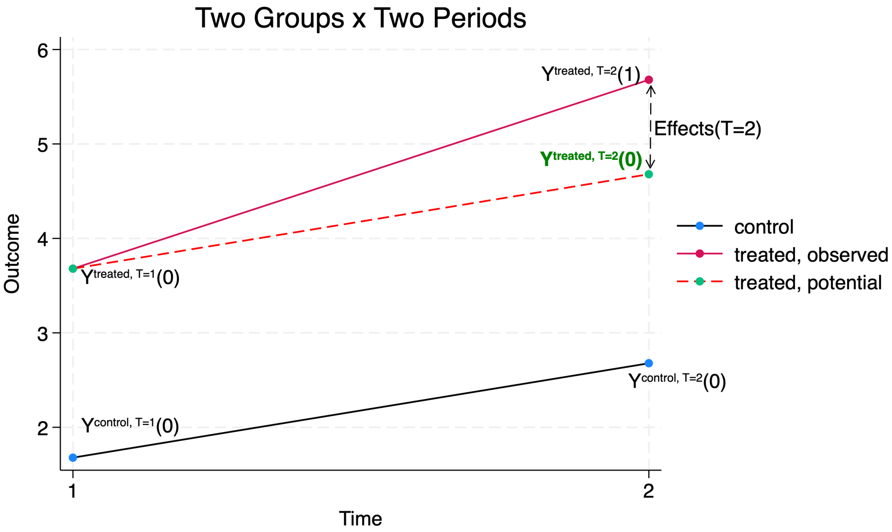
Setup

- Estimate treatment effects using panel data or repeated cross-section
- Treatments may start at different times
- Staggered treatment (once treated, always treated)

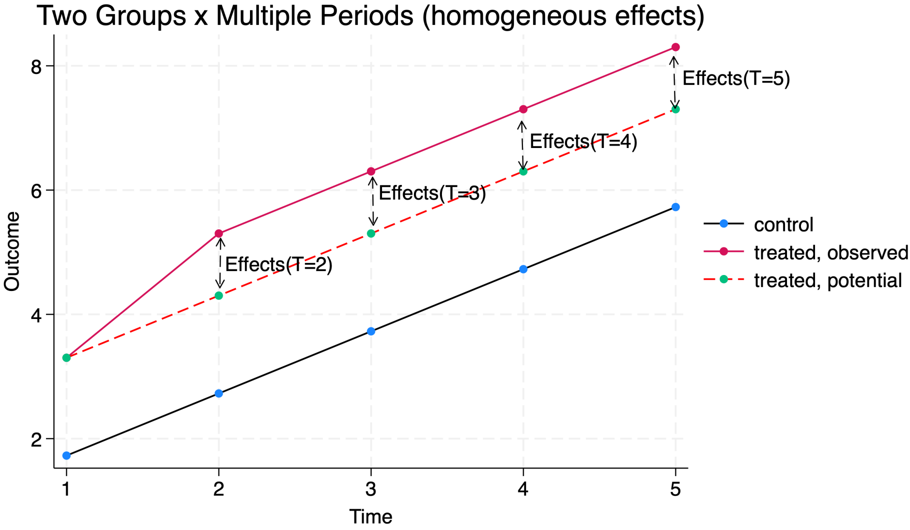
Group/Cohort: units in the same group start the treatment at the same time, different groups start treatments at different times

Objective: Estimate treatment effects for the treated groups

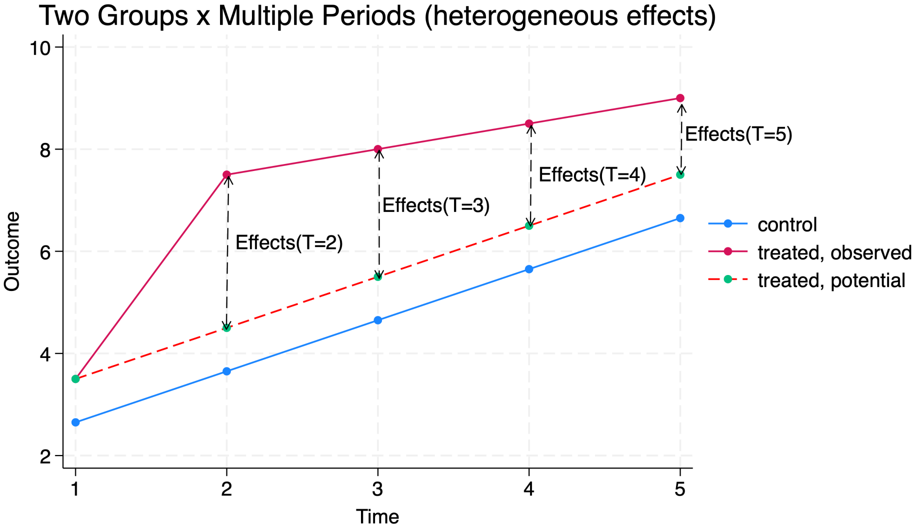
Canonical DID



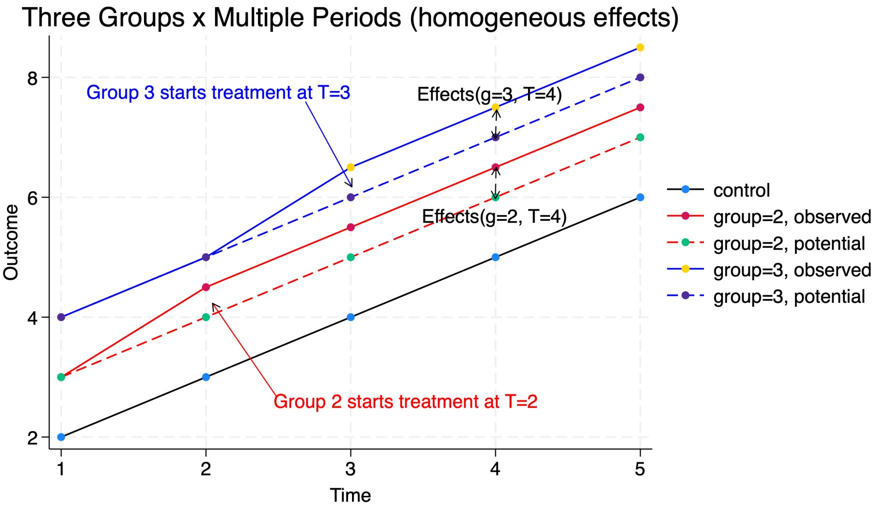
Homogeneous effects across time



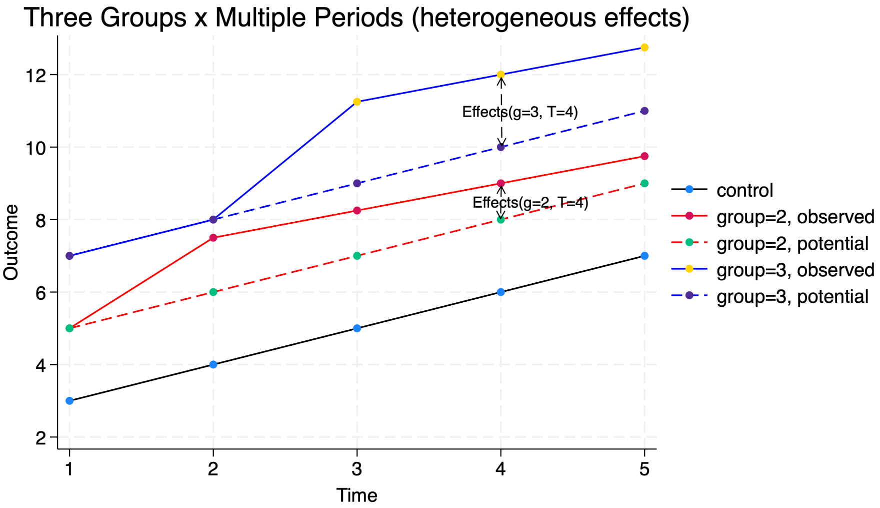
Heterogeneous effects across time



Homogeneous effects across groups and time



Heterogeneous effects across groups and time



Problematic TWFE

$$y_{it} = \theta_t + \eta_i + d_{it}\alpha + v_{it}$$

But,

$$\alpha = \sum w_k Good\_DID_k + \sum w_j Bad\_DID_j$$

1. Newly treated relative to the control group (good)
2. Newly treated relative to the not-yet treated group (good)
3. Newly treated relative to already treated group (bad)

When is TWFE good?

1. There are only two periods
2. The treatment effects are homogenenous across both groups and time

Overview of heterogeneous DID in Stata 18

Estimation:

1. `xthdidregress` and `hdidregress` for panel data and repeated cross-section data
2. Four estimators: `ra`, `ipw`, `aipw` in Callaway and Sant'Anna (2021) and `twfe` in Wooldridge (2021)

Post-estimation:

1. `estat atetplot`: visualize ATETs
2. `estat aggregation`: aggregate ATETs along different dimensions
3. `estat ptrends`: pre-treatment parallel trend tests
4. `estat sci`: simultaneous CI for RA, IPW, and AIPW estimators

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Increasing minimum wage and young employment

- **Outcome**: county-level employment for young workers
- **Treatment**: minimum wage restrictions introduced by State government; see Callaway and Sant'Anna (2021)
- **Multiple periods**: 2002 - 2007 (6 years)
- **Multiple treatment timings**: 2004, 2006, 2007

`xthdidregress aipw`

Define covariates

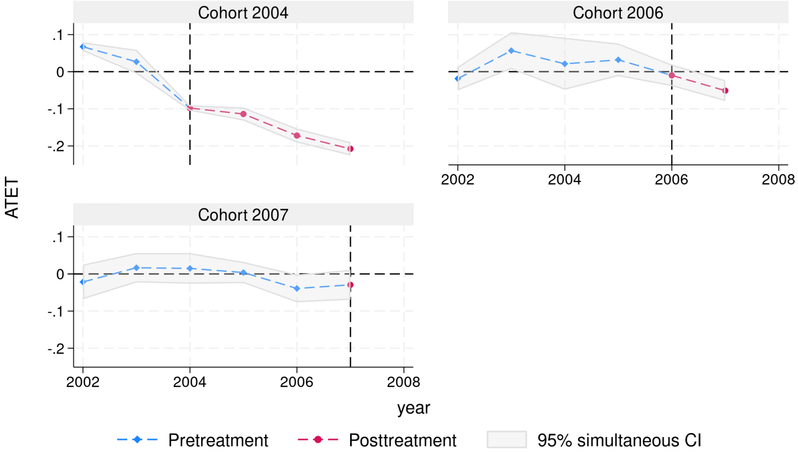
```
global covars i.region pop medinc white hs pov ///
       c.pop#c.pop c.medinc#c.medinc
```

Use AIPW estimator

```
xthdidregress aipw (lemp $covars) (treat $covars), group(state)
```

- Adding covariates for conditional parallel trend
- There are 18 ATET(g,t)'s (6 years $\times$ 3 cohorts)
- Standard errors are adjusted by clusters of state

`estat atetplot, sci`



- Specify option `sci` for simultaneous confidence intervals
- For cohorts 2004 and 2006, minimum wage restriction decreases the employment rate for young workers

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Summarize ATET(g,t)'s

- How do the ATETs vary with the length of exposure to the treatment? (event study)
- How do the ATETs vary with cohorts? (does start treatment earlier matter?)
- How do the ATETs vary with time? (Good year vs. lousy year)
- Overall ATETs across time and cohorts

We can express the aggregations as a weighted mean of all ATETs

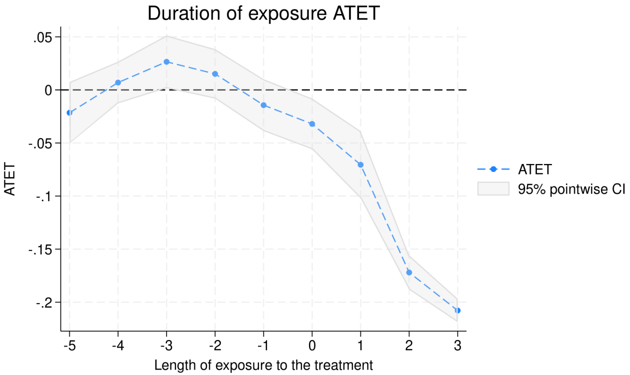
$$\theta = \sum_{g \in \mathbf{G}} \sum_{t=2}^{T} \underbrace{w(g,t)}_{\text{weight}} ATET(g,t)$$

Event study

- Let $e = t - g$ be the length of exposure to the treatment.

$$\theta(\mathbf{e}) = \sum_{g \in \mathbf{G}} \underbrace{\mathbb{1}(g + e \leq T)\,\mathrm{Pr}(G = g | g + e \leq T)}_{\text{propotions used to estimate ATET(g, g+e)}} ATET(g, g + e)$$

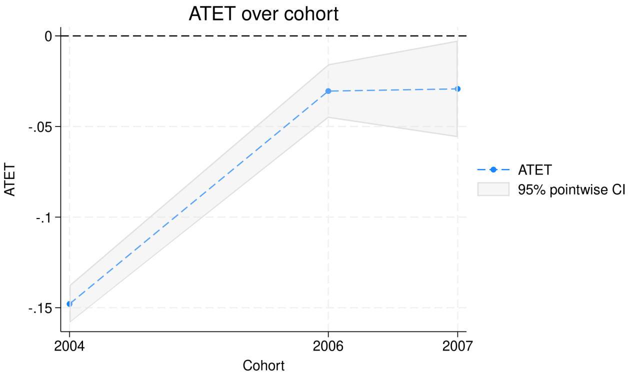
- `estat aggregation,` **dynamic** `graph`



ATETs over cohort

$$\theta(\mathbf{g}) = \sum_{t=g}^{T} \underbrace{\Pr(G = g, T = t | G = g, t \geq g)}_{\text{proportions used to estimate post-treatment ATET(g,t)}} \quad ATET(g, t)$$

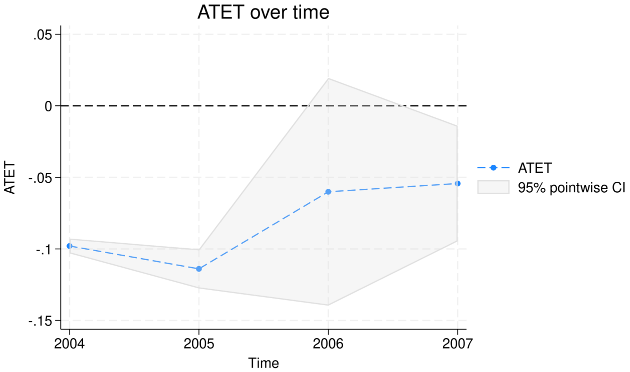
- `estat aggregation,` **cohort** `graph`



ATETs across time

$$\theta(\mathbf{t}) = \sum_{g \in \mathbf{G}} \mathbb{1}(t \geq g) \Pr(G = g | G \leq t) ATET(g,t)$$

- `estat aggregation,` **time** `graph`



Overall aggregations

- A single number to summarize ATET's

$$\theta = \frac{1}{\kappa} \sum_{g \in \mathbf{G}} \sum_{t=2}^{T} \mathbb{1}(t \geq g)\,\mathrm{Pr}(G = g | G \leq T) ATET(g, t)$$

- `estat aggregation,` **overall**

```
. estat aggreg, overall
Overall ATET                                        Number of obs = 15,988
                                    (Std. err. adjusted for 29 clusters in state)
```

lemp	ATET	Robust std. err.	z	P>|z|	[95% conf.	interval]
treat (1 vs 0)	-.062811	.0256879	-2.45	0.014	-.1131582	-.0124637

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Potential outcome framework

Some notations:

- $y_{i,t}(g)$ is unit i's potential outcome at time t if it starts treatment at time g
- $y_{i,t}(\infty)$ is unit i's potential outcome at time t if it is never treated
- G_i indicates unit i's cohort (when the treatment starts), and it is one element in $\mathbf{G} = \{2, \ldots, G, \infty\}$
- $y_{i,t}$ is unit i's observed outcome at time t

$$y_{i,t} = \underbrace{\mathbb{1}(t < G_i) y_{i,t}(\infty)}_{\text{before treatment}} + \underbrace{\mathbb{1}(t \geq G_i) y_{i,t}(G_i)}_{\text{after treatment}}$$

Heterogeneous ATETs

$$ATET(g,t) = \mathbf{E}[y_{i,t}(g) - y_{i,t}(\infty) | G_i = g]$$

Remarks

- $ATET(g,t)$ is a function of two arguments: cohort g and time t
- *ATETs* can be heterogeneous over cohorts, across time, across both time and cohorts
- Objective: consistently estimate ATETs and summarize them

Key assumptions

- Observe I.I.D samples of $\{y_{i,t}, \mathbf{x}_{i,t}, \mathbf{z}_{i,t}, d_{i,t}\}_{i=1,t=1}^{i=N,t=T}$, where $\mathbf{x}_{i,t}$ and $\mathbf{z}_{i,t}$ are covariates, and $d_{i,t}$ is observational level treatment indicator
- No one is treated in the first period
- No anticipation in pre-treatment periods $t < g$

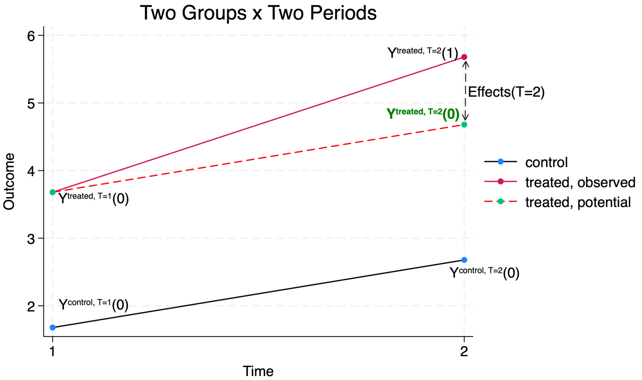
$$\mathbf{E}[y_{i,t}(g)|\mathbf{x}, G_i = g] = \mathbf{E}[y_{i,t}(\infty)|\mathbf{x}, G_i = g]$$

- Conditional parallel trend

$$\mathbf{E}[y_{i,t}(\infty) - y_{i,t-1}(\infty)|\mathbf{x}, G_i = g] = \mathbf{E}[y_{i,t}(\infty) - y_{i,t-1}(\infty)|\mathbf{x}, G_i = \infty]$$

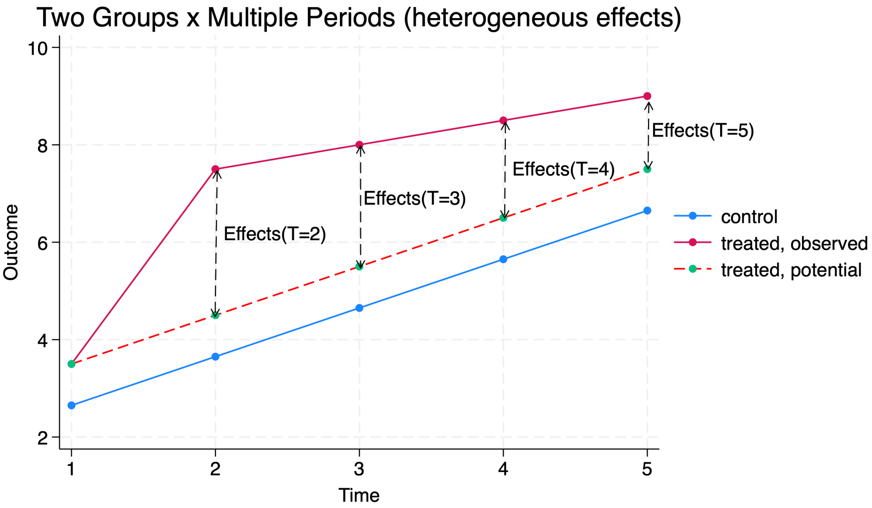
- Overlap assumption for propensity scores

Canonical DID

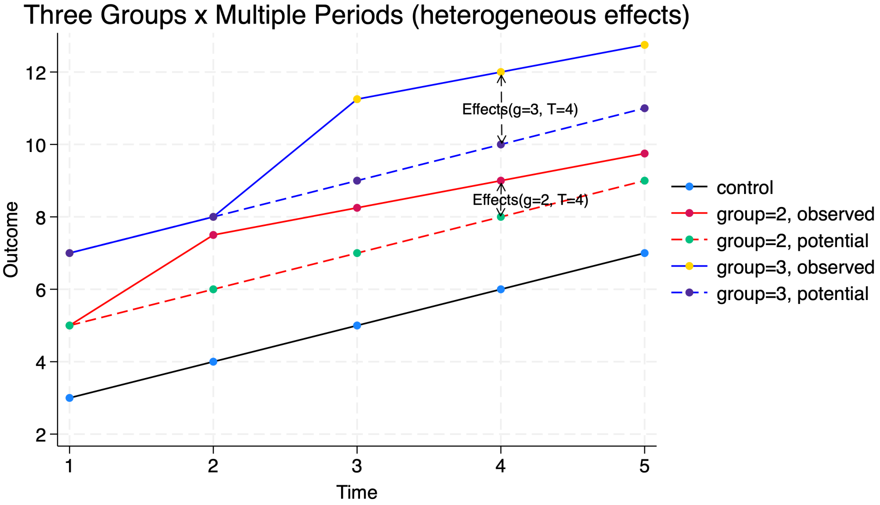


$$
\begin{aligned}
Effects &= \left[Y^{treated,T=2}(1) - Y^{treated,T=1}(0)\right] - \left[Y^{control,T=2}(0) - Y^{control,T=1}(0)\right] \\
&= \underbrace{\left[Y^{treated,T=2} - Y^{treated,T=1}\right]}_{\text{treated differences}} - \underbrace{\left[Y^{control,T=2} - Y^{control,T=1}\right]}_{\text{untreated differences}}
\end{aligned}
$$

Heterogenous DID: benchmark time $g - 1$



$ATET(g, t)$ is reduced to 2×2 DID



Regression adjustment (RA)

$$
\begin{aligned}
ATET(g,t) &= \mathbf{E}\left[\frac{K_g}{\mathbf{E}(K_g)}\left(y_t - y_{g-1} - m_{g,t}\right)\right] \\
&= \underbrace{\mathbf{E}\left[y_t - y_{g-1} | K_g = 1\right]}_{\text{treated differences}} - \underbrace{\mathbf{E}\left[m_{g,t}(\mathbf{x}) | K_g = 1\right]}_{\text{untreated differences}}
\end{aligned}
$$

where

- $K_g = \mathbb{1}(G_i = g)$ and $m_{g,t}(\mathbf{x}) = \mathbf{E}(y_t - y_{g-1} | \mathbf{x}, G_i = \infty)$
- It is 2×2 difference-in-differences (two groups $\times$ two periods)
- Benchmark time: one period before treatment ($g - 1$)
- Benchmark group: never-treated group ($G_i = \infty$)

In Stata, we type

- `xthdidregress ra` $\underbrace{(y \quad \mathbf{x})}_{m_{g,t}(\mathbf{x})}$ `(d), group(id)`

Inverse probability weighting (IPW)

$$ATET(g,t) = \mathbf{E}\left[\left(\frac{K_g}{\mathbf{E}(K_g)} - \frac{\frac{p_g(\mathbf{z})K_\infty}{1-p_g(\mathbf{z})}}{\mathbf{E}\left[\frac{p_g(\mathbf{z})K_\infty}{1-p_g(\mathbf{z})}\right]}\right)(Y_t - Y_{g-1})\right]$$

where

- $p_g(\mathbf{z}) = \Pr(K_g = 1|\mathbf{z}, K_g + K_\infty = 1) = \frac{\Pr(K_g=1|\mathbf{z})}{\Pr(K_g+K_\infty=1|\mathbf{z})}$
- $\frac{p_g(\mathbf{z})}{1-p_g(\mathbf{z})} = \frac{\Pr(K_g=1|\mathbf{z})}{\Pr(K_\infty=1|\mathbf{z})}$. Thus, in the benchmark group (never treated), attach more weights to observations that are more probably observed in the cohort g
- We estimate $p_g(\mathbf{z})$ by a logit regression

In Stata, we type

- `xthdidregress ipw (y)` $\underbrace{(d\ \mathbf{z})}_{p_g(\mathbf{z})}$`, group(id)`

Augmented inverse probability weighting (AIPW)

$$ATET(g,t) = \mathbf{E}\left[\underbrace{\left(\frac{K_g}{\mathbf{E}(K_g)} - \frac{\frac{p_g(\mathbf{z})K_\infty}{1-p_g(\mathbf{z})}}{\mathbf{E}\left[\frac{p_g(\mathbf{z})K_\infty}{1-p_g(\mathbf{z})}\right]} \right) (Y_t - Y_{g-1} -}_{\text{IPW}} \overbrace{m_{g,t}(\mathbf{x})}^{\text{augmented term}}) \right]$$

- AIPW is doubly robust: only one of the outcome model or the treatment model needs to be correctly specified

In Stata, we type

- `xthdidregress aipw` $\underbrace{(y\ \ \mathbf{x})}_{m_{g,t}(\mathbf{x})}$ $\underbrace{(d\ \ \mathbf{z})}_{p_g(\mathbf{z})}$`, group(id)`

TWFE

Traditional TWFE:

$$y_{it} = \theta_t + \eta_i + d_{it}\alpha + v_{it}$$

TWFE in Wooldridge (2021):

$$y_{it} = \theta_t + \eta_i + \sum_{g \in \mathbf{G}} \sum_{s=g}^{T} \alpha_{g,t} \mathbb{1}(G_i = g, t = s) + v_{it}$$

With covariates **x**, add full interactions with θ_t, η_i, and $\mathbb{1}(G_i = g, t = s)$.

In Stata, we type

- `xthdidregress twfe` $\underbrace{(y\ \ \mathbf{x})}_{\textit{TWFE outcome}}$ `(d), group(id)`

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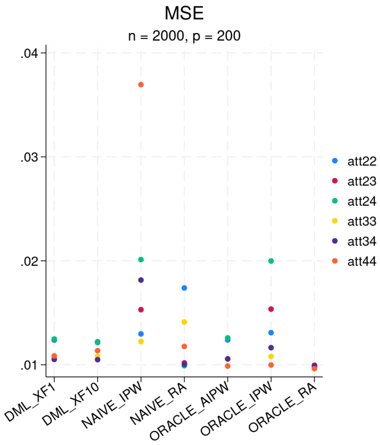
Double machine learning AIPW estimator

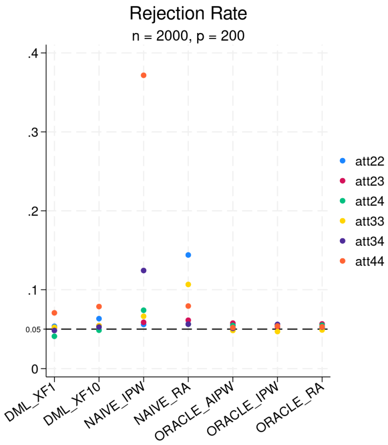
- Recall the AIPW estimator

$$ATET(g,t) = \mathbf{E}\left[\left(\frac{K_g}{\mathbf{E}(K_g)} - \frac{\frac{p_g(\mathbf{z})K_\infty}{1-p_g(\mathbf{z})}}{\mathbf{E}\left[\frac{p_g(\mathbf{z})K_\infty}{1-p_g(\mathbf{z})}\right]}\right)(Y_t - Y_{g-1} - m_{g,t}(\mathbf{x}))\right]$$

- AIPW is not only doubly robust but also Neyman orthognal
- Allowing high-dimensional covariates in $p_g(\mathbf{z})$ and $m_{g,t}(\mathbf{x})$
- Combining the cross-fitting with the AIPW scores
- For details, see the working paper Callaway, Drukker, Liu, and Sant'Anna (2023)

Simulations with high-dimensional covariates





- Double machine learning AIPW (DML_XF1, DML_XF10)
- Naive estimators for IPW and RA
- Oracle estimators for RA, IPW, and AIPW

Summary

- We illustrate the heterogeneous treatment effects across time and groups
- Traditional TWFE is inconsistent
- `xthdidregress` and `hdidregress` implements four estimators to remedy the heterogeneity issues
- `estat atetplot` visualize the heterogeneity at the group-time level (zoom in)
- `estat aggregation` estimate and visualize the heterogeneity at a higher level (zoom out)

References

Callaway, B., D. Drukker, D. Liu, and P. Sant'Anna. 2023. Double/Debiased Machine-learning estimator for Difference-in-Difference with Multiple Periods. URL https://www.doi.org/10.13140/RG.2.2.33815.65447.

Callaway, B., and P. H. Sant'Anna. 2021. Difference-in-Differences with multiple time periods. *Journal of Econometrics* 225: 200–230.

Wooldridge, J. M. 2021. Two-Way Fixed Effects, the Two-Way Mundlak Regression, and Difference-in-Differences Estimators. URL https://ssrn.com/abstract=3931952.

Appendix

Contents

1 xthdidregress

```
. global covars i.region pop medinc white hs pov c.pop#c.pop c.medinc#c.medinc

. xthdidregress aipw (lemp $covars) (treat $covars), group(state)
note: variable _did_cohort, containing cohort indicators formed by treatment
      variable treat and group variable state, was added to the dataset.

Computing ATET for each cohort and time:
Cohort 2004 (6): ...... done
Cohort 2006 (6): ...... done
Cohort 2007 (6): ...... done

Treatment and time information

Time variable: year
Time interval: 2001 to 2007
Control:       _did_cohort = 0
Treatment:     _did_cohort > 0
```

	_did_cohort
Number of cohorts	4
Number of obs	
Never treated	9639
2004	700
2006	1561
2007	4088

```
Heterogeneous-treatment-effects regression          Number of obs    = 15,988
                                                    Number of panels =     29
Estimator:       Augmented IPW
Panel variable:  countyreal
Treatment level: state
Control group:   Never treated

                                 (Std. err. adjusted for 29 clusters in state)
```

Cohort	ATET	Robust std. err.	z	P>|z|	[95% conf.	interval]
2004						
year						
2002	.0672458	.0061125	11.00	0.000	.0552655	.079226
2003	.0266718	.0122508	2.18	0.029	.0026608	.0506829
2004	-.0979371	.002649	-36.97	0.000	-.103129	-.0927451
2005	-.1139248	.0070092	-16.25	0.000	-.1276627	-.1001869
2006	-.1719979	.0082852	-20.76	0.000	-.1882366	-.1557592
2007	-.2078132	.0056814	-36.58	0.000	-.2189485	-.196678
2006						
year						
2002	-.0186685	.0105915	-1.76	0.078	-.0394274	.0020904
2003	.056737	.0181748	3.12	0.002	.0211151	.0923589
2004	.0212315	.0363779	0.58	0.559	-.0500679	.092531
2005	.0319911	.0158191	2.02	0.043	.0009863	.0629959
2006	-.009851	.0117487	-0.84	0.402	-.0328781	.013176
2007	-.0510452	.0092241	-5.53	0.000	-.069124	-.0329664

2007						
year						
2002	-.0215125	.014779	-1.46	0.145	-.0504788	.0074538
2003	.0167167	.0132905	1.26	0.208	-.0093322	.0427655
2004	.0149363	.0133763	1.12	0.264	-.0112809	.0411534
2005	.0038453	.0092391	0.42	0.677	-.014263	.0219537
2006	-.0390546	.0114977	-3.40	0.001	-.0615896	-.0165196
2007	-.0292338	.0136042	-2.15	0.032	-.0558976	-.00257

Note: ATET computed using covariates.

2 estat aggregation, dynamic

```
. estat aggreg, dynamic graph(name(d1))
```

Duration of exposure ATET   Number of obs = 15,988

(Std. err. adjusted for 29 clusters in state)

Exposure	ATET	Robust std. err.	z	P>|z|	[95% conf.	interval]
-5	-.0215125	.014779	-1.46	0.145	-.0504788	.0074538
-4	.0069386	.0100519	0.69	0.490	-.0127627	.0266399
-3	.0264872	.0126915	2.09	0.037	.0016122	.0513621
-2	.0151101	.0118987	1.27	0.204	-.0082109	.0384311
-1	-.0143403	.0124878	-1.15	0.251	-.0388159	.0101353
0	-.032043	.0122219	-2.62	0.009	-.0559975	-.0080885
1	-.0705126	.0161956	-4.35	0.000	-.1022553	-.0387699
2	-.1719979	.0082852	-20.76	0.000	-.1882366	-.1557592
3	-.2078132	.0056814	-36.58	0.000	-.2189485	-.196678

Note: Exposure is the number of periods since the first treatment time.

3 estat aggregation, cohort

```
. estat aggreg, cohort graph(name(c1))
```

ATET over cohort   Number of obs = 15,988

(Std. err. adjusted for 29 clusters in state)

Cohort	ATET	Robust std. err.	z	P>|z|	[95% conf.	interval]
2004	-.1479183	.0053113	-27.85	0.000	-.1583283	-.1375082
2006	-.0304481	.0075561	-4.03	0.000	-.0452578	-.0156384
2007	-.0292338	.0136042	-2.15	0.032	-.0558976	-.00257

4 estat aggregation, time

```
. estat aggreg, time graph(name(t1))
```

ATET over time Number of obs = 15,988

(Std. err. adjusted for 29 clusters in state)

Time	ATET	Robust std. err.	z	P>|z|	[95% conf.	interval]
2004	-.0979371	.002649	-36.97	0.000	-.103129	-.0927451
2005	-.1139248	.0070092	-16.25	0.000	-.1276627	-.1001869
2006	-.0600513	.0406199	-1.48	0.139	-.1396648	.0195622
2007	-.0542855	.0206191	-2.63	0.008	-.0946981	-.0138728